

Pentaquark Decay Amplitudes From $SU(3)$ Flavor Symmetry

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The experimental signature of an $S = +1$ baryon of mass 1540 MeV and width $\sim 10 - 25$ MeV has led to speculation that this is an exotic baryon consisting of 4 quarks and 1 antiquark lying at the top of a $\overline{10}$ $SU(3)_F$ multiplet. The observed decay modes are $\Theta^+ \rightarrow K^+ n$ and $\Theta^+ \rightarrow K_S^0 p$ or in terms of flavor multiplets, $\overline{10} \rightarrow 8 \otimes 8$. The effective Hamiltonian for this decay can be written as a flavor singlet piece plus an $SU(3)$ breaking term. By using the Wigner-Eckart theorem, we then relate the decay amplitudes of all 10 members of the antidecuplet in terms of 5 parameters. Similarly, we perform the same calculation for the proposed exotic multiplets 27 and 35.

I. MOTIVATION

The observed decay modes of $\Theta^+ \rightarrow K^+ n$ [1, 2, 3, 4, 5, 6] and $\Theta^+ \rightarrow K_S^0 p$ [7, 8, 9, 10, 11] have $S = +1$ and baryon number $B = +1$. As such, an ordinary 3 quark baryon cannot be identified with the initial state. Instead, we are led to consider the possibility of an exotic baryon consisting of 4 quarks and 1 antiquark, which in this case would be an $\overline{5}$. The observed state has zero isospin and positive charge, so it may be part of an $I = 1$ isomultiplet. Searches for higher isopartners, however, have thus far been negative. All the data therefore points to this being an $I = 0$ exotic baryon with $S = +1$, and $SU(3)_F$ gives the antidecuplet $\overline{10}$ as a probable candidate for its flavor multiplet. However the possibility that the state actually belongs to the 27 or 35 multiplet has not yet been ruled out. There has been much activity in explaining the observed resonance in chiral soliton models [12, 13, 14, 15, 16, 17] and the diquark model [18]. Work has also been done in exploiting the $SU(3)_F$ symmetry among the light quarks to determine pentaquark decay amplitudes [19, 20, 21]. Here we adopt a similar approach to the latter, but include $SU(3)_F$ breaking in the analysis.

$SU(3)_F$ has proven to be a very accurate symmetry in the baryon sector of QCD bound states due to the small mass differences between the u, d, and s quark masses relative to Λ_{QCD} . $SU(3)_F$ breaking can be incorporated through the T^8 Gell-Mann matrix which treats the s quark differently than the u and d quarks, and if further accuracy is required, isospin breaking can be generated via the T^3 matrix.

II. THE EFFECTIVE HAMILTONIAN

If, in fact, the observed decay mode is that of an exotic baryon lying at the top of an antidecuplet, then we can parametrize all the decay modes of its members by using $SU(3)_F$ symmetry and incorporating $SU(3)_F$ breaking to first order in our effective Hamiltonian:

$$H = \alpha \mathbb{1} + \mathcal{O}_j^i [T^8]_i^j \quad (1)$$

where we have explicitly shown the $SU(3)_F$ transformation properties of each term in the Hamiltonian. Since Isospin splittings are of the order of a few MeV, we have neglected an $SU(2)$ breaking term in our Hamiltonian corresponding to this effect.

We wish to calculate the matrix element:

$$\langle 8 \otimes 8 | H | \overline{10} \rangle \quad (2)$$

which will yield the amplitude for a pentaquark to decay into a pseudoscalar meson and octet baryon.

According to the Wigner-Eckart theorem, we can separate out the dynamical and group theoretical pieces of the matrix element into a reduced matrix element in which the dynamics is encoded, and a matrix element that depends solely upon the $SU(3)$ properties of the states and Hamiltonian. This leads to:

$$\langle 8 \otimes 8 | H | \overline{10} \rangle = \alpha \langle j | \langle l |^{rst} \rangle (B^*)_i^j (M^*)_k^l D_{rst} + \langle j | \langle l | \mathcal{O}_n^m |^{rst} \rangle (B^*)_i^j (M^*)_k^l [T^8]_m^n D_{rst} \quad (3)$$

where we have represented the baryon octet, pseudoscalar meson octet, and antidecuplet states respectively as:

$$| 8 \rangle = \langle j | (B^*)_i^j$$

$$\begin{aligned}
\langle 8| &= \langle_l^k |(M^*)_k^l \\
|\overline{10}\rangle &= D_{rst} |rst\rangle
\end{aligned}
\tag{4}$$

with the baryon octet tensor coefficients:

$$\begin{aligned}
B_3^1 &= P, & B_3^2 &= N, & B_2^1 &= \Sigma^+, \\
B_1^1 &= \frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda^0}{\sqrt{6}}, & B_2^2 &= -\frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda^0}{\sqrt{6}}, & B_3^3 &= -\frac{2\Lambda^0}{\sqrt{6}}, \\
B_1^2 &= \Sigma^-, & B_2^3 &= \Xi^0, & B_1^3 &= \Xi^-.
\end{aligned}
\tag{5}$$

The meson tensor coefficients are:

$$\begin{aligned}
M_3^1 &= K^+, & M_3^2 &= K^0, & M_2^1 &= \pi^+, \\
M_1^1 &= \frac{\pi^0}{\sqrt{2}} + \frac{\eta^0}{\sqrt{6}}, & M_2^2 &= -\frac{\pi^0}{\sqrt{2}} + \frac{\eta^0}{\sqrt{6}}, & M_3^3 &= -\frac{2\eta^0}{\sqrt{6}}, \\
M_1^2 &= \pi^-, & M_2^3 &= \bar{K}^0, & M_1^3 &= K^-.
\end{aligned}
\tag{6}$$

and the antidecuplet has tensor coefficients:

$$\begin{aligned}
D_{333} &= \Theta_{\overline{10}}^+, & D_{233} &= \frac{P_{\overline{10}}}{\sqrt{3}}, & D_{133} &= \frac{N_{\overline{10}}}{\sqrt{3}}, \\
D_{223} &= \frac{\Sigma_{\overline{10}}^+}{\sqrt{3}}, & D_{123} &= \frac{\Sigma_{\overline{10}}^0}{\sqrt{6}}, & D_{113} &= \frac{\Sigma_{\overline{10}}^-}{\sqrt{3}}, \\
D_{222} &= \Xi_{\overline{10}}^+, & D_{122} &= \frac{\Xi_{\overline{10}}^0}{\sqrt{3}}, & D_{112} &= \frac{\Xi_{\overline{10}}^-}{\sqrt{3}}, \\
D_{111} &= \Xi_{\overline{10}}^{--}
\end{aligned}
\tag{7}$$

The indices of the $\overline{10}$ are completely symmetric and thereby we omitted redundant elements from the tensor above.

III. TENSOR DECOMPOSITION

In order to make usage of our expression for the decay amplitude we need to know what irreducible representations of $SU(3)$ the tensor products $8 \otimes 8$ and $\overline{10} \otimes 8$, where the 8 in the second tensor product corresponds to the $SU(3)$ breaking term in the Hamiltonian, decompose into:

$$\begin{aligned}
8 \otimes 8 &= 1 \oplus 8 \oplus 8 \oplus 10 \oplus \overline{10} \oplus 27 \\
\overline{10} \otimes 8 &= 8 \oplus \overline{10} \oplus 27 \oplus \overline{35}
\end{aligned}
\tag{8}$$

Because there is a $\overline{10}$ in the $8 \otimes 8$ decomposition, the pentaquark $\overline{10}$ will be able to decay into an octet baryon + pseudoscalar meson via the singlet piece of the Hamiltonian. In addition, both $8 \otimes 8$ and $\overline{10} \otimes 8$ contain an 8, $\overline{10}$, and 27, so there will also be contributions from the $SU(3)$ breaking term. As there are two octets in $8 \otimes 8$, the total number of reduced matrix elements will be five, which correspond to one from the singlet and four from the $SU(3)$ breaking term, so we may then calculate all decay amplitudes of the ten members of the antidecuplet in terms of five parameters.

The decay amplitude is a scalar quantity, so all the indices on the tensors must be fully contracted. Because of the orthogonality of different irreducible representations, what indices we contract can be determined in a fairly straightforward fashion [22]. The explicit form for the 8, $\overline{10}$, and 27 tensors from each tensor product is then necessary. The fundamental form of each of these for $8 \otimes 8$ are:

$$\begin{aligned}
(8_a)_j^i &= B_m^i M_j^m - \frac{1}{3} \delta_j^i B_m^n M_n^m \\
(8_b)_j^i &= B_j^m M_m^i - \frac{1}{3} \delta_j^i B_m^n M_n^m
\end{aligned}$$

$$\begin{aligned}
\overline{10}_{ijk} &= \epsilon_{irs}(B_j^r M_k^s + B_k^r M_j^s) + \epsilon_{jrs}(B_k^r M_i^s + B_i^r M_k^s) + \epsilon_{krs}(B_i^r M_j^s + B_j^r M_i^s) \\
27_{jl}^{ik} &= (B_j^i M_l^k + B_l^i M_j^k + B_j^k M_l^i + B_l^k M_j^i) - \frac{1}{5}[\delta_l^i(B_j^m M_m^k + B_m^k M_j^m) \\
&\quad + \delta_j^i(B_l^m M_m^k + B_m^k M_l^m) + \delta_l^k(B_j^m M_m^i + B_m^i M_j^m) + \delta_j^k(B_l^m M_m^i + B_m^i M_l^m)] \\
&\quad + \frac{1}{10}[(\delta_l^i \delta_j^k + \delta_j^i \delta_l^k)B_n^m M_m^n]
\end{aligned} \tag{9}$$

There are two different expressions for 8 because it appears twice in the tensor decomposition of Eq. 8. The same irreducible representations appearing in the tensor decomposition of $\overline{10} \otimes 8$ are:

$$\begin{aligned}
8_j^i &= \epsilon^{irs}[T^8]_r^n D_{nsj} \\
\overline{10}_{ijk} &= [T^8]_i^n D_{njk} + [T^8]_j^n D_{nik} + [T^8]_k^n D_{nji} \\
27_{jl}^{ik} &= [\epsilon^{irs}[T^8]_r^k D_{sjl} + \epsilon^{krs}[T^8]_r^i D_{sjl}] - \frac{1}{5}[\delta_l^i \epsilon^{krs}[T^8]_r^m D_{sjm} + \delta_j^i \epsilon^{krs}[T^8]_r^m D_{slm} \\
&\quad + \delta_j^k \epsilon^{irs}[T^8]_r^m D_{slm} + \delta_l^k \epsilon^{irs}[T^8]_r^m D_{sjm}]
\end{aligned} \tag{10}$$

The $\overline{10}$ and 27 have the required symmetry among their upper and lower indices respectively, and the 8 and 27 tensors are also traceless as is necessary.

IV. THE $\overline{10}$ DECAY AMPLITUDE

The decay amplitude may now be evaluated by contracting the irreducible tensors from $8 \otimes 8$ with the corresponding tensor from $\overline{10} \otimes 8$. There is a unique contraction for each irreducible representation. The tensor coefficients of a bra state vector are the complex-conjugate transpose of the tensor coefficients of the ket state vector, for example:

$$|\overline{10}\rangle = D_{rst}|^{rst}\rangle \rightarrow \langle^{rst}|(D^*)_{rst} = \langle\overline{10}| \tag{11}$$

In terms of reduced matrix elements, the decay amplitude is:

$$\begin{aligned}
\langle 8 \otimes 8 | H | \overline{10} \rangle &= \alpha_{\overline{10}} * (\overline{10}^*)_{lmn} D_{lmn} + \beta_{\overline{10}} * (\overline{10}^*)_{lmn} \overline{10}_{lmn} + \gamma_{\overline{10}} * (8_a^*)_m^n 8_m^n \\
&\quad + \delta_{\overline{10}} * (8_b^*)_m^n 8_m^n + \epsilon_{\overline{10}} * (27^*)_{kl}^{mn} 27_{kl}^{mn}
\end{aligned} \tag{12}$$

where the greek letters are the reduced matrix elements and the first irreducible representation in each term corresponds to the $8 \otimes 8$ tensors defined in Eq. 9, whereas the second tensor corresponds to those of $\overline{10} \otimes 8$ defined in Eq. 10. Again there are two terms involving the octets as there are two octets in the $8 \otimes 8$ decomposition. To emphasize that this quantity is an $SU(3)$ scalar, we could have written it in terms of tensor coefficients \overline{B} and \overline{M} where:

$$\overline{B}_j^i = (B^*)_i^j \tag{13}$$

which holds component wise. It is perhaps more transparent to use the RHS of the equality as we have explicit forms for these tensors in Eq. 9.

The first term in Eq. 12 corresponds to the singlet piece in the Hamiltonian of Eq. 3, and is the dominant contribution to the amplitude. The four remaining terms are all from the $SU(3)$ breaking term in the Hamiltonian, and as such, one expects them to be on the order of one-third the strength of the singlet contribution. The coefficients of each reduced matrix element for all the decay modes of the $\overline{10}$ into an octet baryon + pseudoscalar meson are tabulated in Appendix B. The particle assignments are given in Appendix A.

V. THE 27 DECAY AMPLITUDE

Similarly to the calculation for the $\overline{10}$ decay amplitude outlined above, we may also consider the decays of the postulated 27 exotic multiplet into octet baryons + pseudoscalar mesons. From Eq. 8, we see that there is a 27 in the decomposition of $8 \otimes 8$, so the 27 exotic multiplet will decay into $8 \otimes 8$ via the singlet piece of the Hamiltonian. There will also be contributions from the $SU(3)$ breaking term in the Hamiltonian as we have:

$$27 \otimes 8 = 8 \oplus 10 \oplus \overline{10} \oplus 27 \oplus 27 \oplus 35 \oplus \overline{35} \oplus 64 \tag{14}$$

Referencing this against Eq. 8, we see that there will be six reduced matrix elements for the $SU(3)$ breaking term, each corresponding to the matrix element between the same irreducible representations in the decompositions, and hence a total of seven parameters will govern the decay amplitude of the 27.

The 27 state is:

$$|27\rangle = W_{jl}^{ik} |_{ik}^{jl}\rangle \quad (15)$$

where W_{jl}^{ik} is both traceless and symmetric in its upper and lower indices respectively with elements:

- $S = 1, I = 1$

$$W_{33}^{11} = \Theta_{27}^{++}, \quad W_{33}^{12} = \frac{\Theta_{27}^+}{\sqrt{2}}, \quad W_{33}^{22} = \Theta_{27}^0, \quad (16)$$

- $S = 0, I = 3/2, 1/2$

$$\begin{aligned} W_{23}^{11} &= \frac{\Delta_{27}^{++}}{\sqrt{2}}, & W_{13}^{11} &= \frac{\Delta_{27}^+}{\sqrt{6}} + \frac{2P_{27}}{\sqrt{30}}, & W_{23}^{12} &= -\frac{\Delta_{27}^+}{\sqrt{6}} + \frac{P_{27}}{\sqrt{30}}, & W_{33}^{13} &= -\frac{3P_{27}}{\sqrt{30}}, \\ W_{13}^{12} &= -\frac{\Delta_{27}^0}{\sqrt{6}} + \frac{N_{27}}{\sqrt{30}}, & W_{23}^{22} &= \frac{\Delta_{27}^0}{\sqrt{6}} + \frac{2N_{27}}{\sqrt{30}}, & W_{33}^{23} &= -\frac{3N_{27}}{\sqrt{30}}, & W_{13}^{22} &= \frac{\Delta_{27}^-}{\sqrt{2}}, \end{aligned} \quad (17)$$

- $S = -1, I = 2, 1, 0$

$$\begin{aligned} W_{22}^{11} &= \Sigma_{27}^{'++}, & W_{12}^{11} &= \frac{\Sigma_{27}^{'+}}{2} + \frac{\Sigma_{27}^+}{2\sqrt{5}}, & W_{22}^{12} &= -\frac{\Sigma_{27}^{'+}}{2} + \frac{\Sigma_{27}^+}{2\sqrt{5}}, & W_{23}^{13} &= -\frac{\Sigma_{27}^+}{\sqrt{5}}, \\ W_{11}^{11} &= \frac{\Sigma_{27}^{'0}}{\sqrt{6}} - \frac{\Sigma_{27}^0}{\sqrt{10}} - \frac{\Lambda_{27}^0}{\sqrt{30}}, & W_{12}^{12} &= -\frac{\Sigma_{27}^{'0}}{\sqrt{6}} - \frac{\Lambda_{27}^0}{2\sqrt{30}}, & W_{13}^{13} &= \frac{\Sigma_{27}^0}{\sqrt{10}} + \frac{3\Lambda_{27}^0}{2\sqrt{30}}, \\ W_{22}^{22} &= \frac{\Sigma_{27}^{'0}}{\sqrt{6}} + \frac{\Sigma_{27}^0}{\sqrt{10}} - \frac{\Lambda_{27}^0}{\sqrt{30}}, & W_{23}^{23} &= -\frac{\Sigma_{27}^0}{\sqrt{10}} + \frac{3\Lambda_{27}^0}{2\sqrt{30}}, & W_{33}^{33} &= -\frac{3\Lambda_{27}^0}{\sqrt{30}}, \\ W_{11}^{12} &= \frac{\Sigma_{27}^{' -}}{2} + \frac{\Sigma_{27}^-}{2\sqrt{5}}, & W_{12}^{22} &= -\frac{\Sigma_{27}^{' -}}{2} + \frac{\Sigma_{27}^-}{2\sqrt{5}}, & W_{13}^{23} &= -\frac{\Sigma_{27}^-}{\sqrt{5}}, & W_{11}^{22} &= \Sigma_{27}^{' - -}, \end{aligned} \quad (18)$$

- $S = -2, I = 3/2, 1/2$

$$\begin{aligned} W_{22}^{13} &= \frac{\Xi_{27}^{'+}}{\sqrt{2}}, & W_{12}^{13} &= \frac{\Xi_{27}^{'0}}{\sqrt{6}} + \frac{\Xi_{27}^0}{\sqrt{30}}, & W_{22}^{23} &= -\frac{\Xi_{27}^{'0}}{\sqrt{6}} + \frac{2\Xi_{27}^0}{\sqrt{30}}, & W_{23}^{33} &= -\frac{3\Xi_{27}^0}{\sqrt{30}}, \\ W_{11}^{13} &= \frac{\Xi_{27}^{' -}}{\sqrt{6}} + \frac{2\Xi_{27}^-}{\sqrt{30}}, & W_{12}^{23} &= -\frac{\Xi_{27}^{' -}}{\sqrt{6}} + \frac{\Xi_{27}^-}{\sqrt{30}}, & W_{13}^{33} &= -\frac{3\Xi_{27}^-}{\sqrt{30}}, & W_{11}^{23} &= \frac{\Xi_{27}^{' - -}}{\sqrt{2}}, \end{aligned} \quad (19)$$

- $S = -3, I = 1$

$$W_{22}^{33} = \Omega_{27}^0, \quad W_{12}^{33} = \frac{\Omega_{27}^-}{\sqrt{2}}, \quad W_{11}^{33} = \Omega_{27}^{--}. \quad (20)$$

where the primed particles are associated with the higher isospin representation in each strangeness multiplet (see Section A for the 27 weight diagram), i.e. for the $S = -1$ multiplet, we have the triple state: $\Sigma_{27}^{'0}$ has $I = 2$, Σ_{27}^0 has $I = 1$, and Λ_{27}^0 has $I = 0$.

The irreducible tensors associated with the decomposition in Eq. 14 are:

$$\begin{aligned} 8_j^i &= [T^8]_n^m W_{mj}^{ni} \\ 10^{ijk} &= \epsilon^{irs} [T^8]_r^t W_{st}^{jk} + \epsilon^{krs} [T^8]_r^t W_{st}^{ij} + \epsilon^{jrs} [T^8]_r^t W_{st}^{ki} \\ \overline{10}_{ijk} &= \epsilon_{irs} [T^8]_t^r W_{jk}^{st} + \epsilon_{krs} [T^8]_t^r W_{ij}^{st} + \epsilon_{jrs} [T^8]_t^r W_{ki}^{st} \\ (27_a)_{jl}^{ik} &= ([T^8]_j^m W_{ml}^{ik} + [T^8]_l^m W_{mj}^{ik}) - \frac{1}{5} (\delta_j^i [T^8]_n^m W_{ml}^{nk} + \delta_l^i [T^8]_n^m W_{mj}^{nk} + \delta_j^k [T^8]_n^m W_{ml}^{ni} + \delta_l^k [T^8]_n^m W_{mj}^{ni}) \\ (27_b)_{jl}^{ik} &= ([T^8]_m^i W_{jl}^{mk} + [T^8]_m^k W_{jl}^{im}) - \frac{1}{5} (\delta_j^i [T^8]_n^m W_{ml}^{nk} + \delta_l^i [T^8]_n^m W_{mj}^{nk} + \delta_j^k [T^8]_n^m W_{ml}^{ni} + \delta_l^k [T^8]_n^m W_{mj}^{ni}) \end{aligned} \quad (21)$$

where we have only given the irreducible representations that are not orthogonal to the representations contained in $8 \otimes 8$. In terms of reduced matrix elements, the decay amplitude then takes the form:

$$\begin{aligned} \langle 8 \otimes 8 | H | 27 \rangle = & \alpha_{27} * (27^*)_{ik}^{jl} W_{ik}^{jl} + \beta_{27} * (8_a^*)_m^n 8_m^n + \gamma_{27} * (8_b^*)_m^n 8_m^n + \delta_{27} * (10^*)^{ijk} 10^{ijk} + \epsilon_{27} * (\overline{10}^*)_{lmn} \overline{10}_{lmn} \\ & + \psi_{27} * (27^*)_{kl}^{mn} (27_a)_{kl}^{mn} + \zeta_{27} * (27^*)_{kl}^{mn} (27_b)_{kl}^{mn} \end{aligned} \quad (22)$$

where as in Eq. 12, the first tensor corresponds to $8 \otimes 8$ with the baryon octet and meson tensor coefficient complex-conjugated and transposed. There are two terms containing octets because the 8 appears twice in the $8 \otimes 8$ decomposition, and there are two terms containing the 27 tensors because, as in Eq. 21, there are two 27 representations appearing in the $27 \otimes 8$ decomposition of Eq. 14. The first term in the amplitude corresponds to the singlet piece of the Hamiltonian and as such is the dominant contribution. The subsequent six terms are all from $SU(3)$ breaking and hence are on the order of one-third the strength of α_{27} . The coefficients of the reduced matrix elements associated with each decay mode of the 27 are summarized in Appendix C, with the particle assignments and associated weight diagram in Appendix A.

VI. THE 35 DECAY AMPLITUDE

The 35 exotic multiplet may also decay into an octet baryon + pseudoscalar meson. Again, $SU(3)$ flavor symmetry will allow us to parametrize all the decay modes in terms of just a few parameters. Since the decomposition of the tensor product $8 \otimes 8$ in Eq. 8 does not contain a 35, there will be no singlet contribution from the effective Hamiltonian in Eq. 1. The leading order contribution will be from the $SU(3)$ breaking term.

Since the $SU(3)$ breaking term in Eq. 1 transforms like an 8, we need the decomposition of $35 \otimes 8$ into irreducible representations in order to find out how many reduced matrix elements there are in the decay amplitude. The decomposition is:

$$35 \otimes 8 = 10 \oplus 27 \oplus 28 \oplus 35 \oplus 35 \oplus 64 \oplus 81 \quad (23)$$

Only the 10 and 27 multiplets in this decomposition have non-vanishing matrix elements with $8 \otimes 8$ by referring back to Eq. 8, thus there will only be two reduced matrix elements in the decay amplitude:

$$\langle 8 \otimes 8 | H | 35 \rangle = \beta_{35} * (10^*)^{lmn} 10^{lmn} + \gamma_{35} * (27^*)_{kl}^{mn} 27_{kl}^{mn} \quad (24)$$

The 35 state is:

$$|35\rangle = F_m^{ijkl} |_{ijkl}^m \rangle \quad (25)$$

where tensor F_m^{ijkl} is traceless and symmetric in its upper indices with coefficients:

- $S = 1, I = 2$

$$F_3^{1111} = \Theta_{35}^{+++}, \quad F_3^{1112} = \frac{\Theta_{35}^{++}}{2}, \quad F_3^{1122} = \frac{\Theta_{35}^+}{\sqrt{6}}, \quad F_3^{1222} = \frac{\Theta_{35}^0}{2}, \quad F_3^{2222} = \Theta_{35}^-, \quad (26)$$

- $S = 0, I = 5/2, 3/2$

$$\begin{aligned} F_2^{1111} &= \Delta_{35}'^{+++}, & F_1^{1111} &= \frac{\Delta_{35}^{++}}{\sqrt{5}} + \frac{2\Delta_{35}^{++}}{\sqrt{30}}, & F_2^{1112} &= -\frac{\Delta_{35}'^{++}}{\sqrt{5}} + \frac{\Delta_{35}^{++}}{2\sqrt{30}}, \\ F_3^{1113} &= -\frac{5\Delta_{35}^{++}}{2\sqrt{30}}, & F_1^{1112} &= \frac{\Delta_{35}^{++}}{\sqrt{10}} + \frac{\Delta_{35}^{++}}{2\sqrt{10}}, & F_2^{1122} &= -\frac{\Delta_{35}'^{+}}{\sqrt{10}} + \frac{\Delta_{35}^{+}}{3\sqrt{10}}, \\ F_3^{1123} &= -\frac{5\Delta_{35}^{+}}{6\sqrt{10}}, & F_1^{1122} &= \frac{\Delta_{35}'^0}{\sqrt{10}} + \frac{\Delta_{35}^0}{3\sqrt{10}}, & F_2^{1222} &= -\frac{\Delta_{35}'^0}{\sqrt{10}} + \frac{\Delta_{35}^0}{2\sqrt{10}}, \\ F_3^{1223} &= -\frac{5\Delta_{35}^0}{6\sqrt{10}}, & F_1^{1222} &= \frac{\Delta_{35}'^-}{\sqrt{5}} + \frac{\Delta_{35}^-}{2\sqrt{30}}, & F_2^{2222} &= -\frac{\Delta_{35}'^-}{\sqrt{5}} + \frac{2\Delta_{35}^-}{\sqrt{30}}, \\ F_3^{2223} &= -\frac{5\Delta_{35}^-}{2\sqrt{30}}, & F_1^{2222} &= \Delta_{35}'^{--} \end{aligned} \quad (27)$$

- $S = -1, I = 2, 1$

$$\begin{aligned}
F_2^{1113} &= \frac{\Sigma_{35}^{'++}}{2}, & F_1^{1113} &= \frac{\Sigma_{35}^{'++}}{4} + \frac{\Sigma_{35}^+}{4}, & F_2^{1123} &= -\frac{\Sigma_{35}^{'++}}{4} + \frac{\Sigma_{35}^+}{12}, & F_3^{1133} &= -\frac{\Sigma_{35}^+}{3}, \\
F_1^{1123} &= \frac{\Sigma_{35}^{'0}}{2\sqrt{6}} + \frac{\Sigma_{35}^0}{6\sqrt{2}}, & F_2^{1223} &= -\frac{\Sigma_{35}^{'0}}{2\sqrt{6}} + \frac{\Sigma_{35}^0}{6\sqrt{2}}, & F_3^{1233} &= -\frac{\Sigma_{35}^0}{3\sqrt{2}}, \\
F_1^{1223} &= \frac{\Sigma_{35}^{'--}}{4} + \frac{\Sigma_{35}^-}{12}, & F_2^{2223} &= -\frac{\Sigma_{35}^{'--}}{4} + \frac{\Sigma_{35}^-}{4}, & F_3^{2233} &= -\frac{\Sigma_{35}^-}{3}, & F_1^{2223} &= \frac{\Sigma_{35}^{'--}}{2}.
\end{aligned} \tag{28}$$

- $S = -2, I = 3/2, 1/2$

$$\begin{aligned}
F_2^{1133} &= \frac{\Xi_{35}^{'++}}{\sqrt{6}}, & F_1^{1133} &= \frac{\Xi_{35}^{'0}}{3\sqrt{2}} + \frac{\Xi_{35}^0}{3\sqrt{2}}, & F_2^{1233} &= -\frac{\Xi_{35}^{'0}}{3\sqrt{2}} + \frac{\Xi_{35}^0}{6\sqrt{2}}, & F_3^{1333} &= -\frac{\Xi_{35}^0}{2\sqrt{2}}, \\
F_1^{1233} &= \frac{\Xi_{35}^{'--}}{3\sqrt{2}} + \frac{\Xi_{35}^-}{6\sqrt{2}}, & F_2^{2233} &= -\frac{\Xi_{35}^{'--}}{3\sqrt{2}} + \frac{\Xi_{35}^-}{3\sqrt{2}}, & F_3^{2333} &= -\frac{\Xi_{35}^-}{2\sqrt{2}}, & F_1^{2233} &= \frac{\Xi_{35}^{'--}}{\sqrt{6}}
\end{aligned} \tag{29}$$

- $S = -3, I = 1, 0$

$$\begin{aligned}
F_2^{1333} &= \frac{\Omega_{35}^{'0}}{2}, & F_1^{1333} &= \frac{\Omega_{35}^{'--}}{2\sqrt{2}} + \frac{\Omega_{35}^-}{2\sqrt{3}}, & F_2^{2333} &= -\frac{\Omega_{35}^{'--}}{2\sqrt{2}} + \frac{\Omega_{35}^-}{2\sqrt{3}}, \\
F_3^{3333} &= -\frac{\Omega_{35}^-}{\sqrt{3}}, & F_1^{2333} &= \frac{\Omega_{35}^{'--}}{2}.
\end{aligned} \tag{30}$$

- $S = -4, I = 1/2$

$$F_2^{3333} = \Phi^-, \quad F_1^{3333} = \Phi^{--}. \tag{31}$$

where the primed greek letters correspond to the higher isospin multiplet for a given strangeness, i.e. for $S = -1$, $\Sigma_{35}^{'0}$ has $I = 2$, and Σ_{35}^0 has $I = 1$. The $SU(3)$ weight diagram for the 35 is shown in Appendix A.

The irreducible tensors associated with the decomposition of Eq. 23 are:

$$\begin{aligned}
10^{ijk} &= [T^8]_n^m F_m^{nijk} \\
27_{jl}^{ik} &= \epsilon_{jst} [T^8]_m^s F_l^{tmik} + \epsilon_{lst} [T^8]_m^s F_j^{tmik}
\end{aligned} \tag{32}$$

We may now evaluate the decay amplitude, Eq. 24, using the irreducible tensors for $8 \otimes 8$ from Eq. 9 and those of Eq. 32. The coefficients of the reduced matrix elements for each decay mode of the 35 are tabulated in Appendix D.

VII. DISCUSSION

The decay amplitudes for all isomultiplets in the $\overline{10}$, 27, and 35 can now be computed by reading off the coefficients corresponding to a particular decay mode tabulated in appendices B, C, and D. One may then calculate the decay width with appropriate spin-averaging factors as:

$$\frac{d\Gamma}{d\Omega} = \frac{|a_{if}|^2}{32\pi^2} \frac{p}{M^2} = \frac{|a_{if}|^2}{64\pi^2} \frac{[M^4 - 2(m_1^2 + m_2^2)M^2 + (m_1^2 - m_2^2)^2]^{1/2}}{M^3} \tag{33}$$

where a_{if} is the decay amplitude, and the two-body phase space factor has been made explicit.

Current experimental data shows a range of values for the width of $\Theta_{\overline{10}}$, and in most cases it falls below the detector resolution. Improvements in the measurement of the width of $\Theta_{\overline{10}}$ as well as measurements of the widths of additional pentaquarks are necessary in order for the decay amplitudes, and hence decay widths and branching ratios, to be determined for all members of the $SU(3)$ multiplet. However, if enough widths are measured, and one is able to obtain values for all the reduced matrix elements corresponding to a given multiplet, then it will be possible to calculate the partial decay widths for all members of the multiplet. One may then compare these results to the various theoretical models and potentially rule some of these out.

VIII. ACKNOWLEDGEMENTS

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APPENDIX A: $SU(3)$ WEIGHT DIAGRAMS

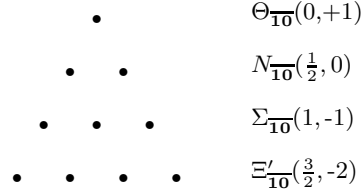
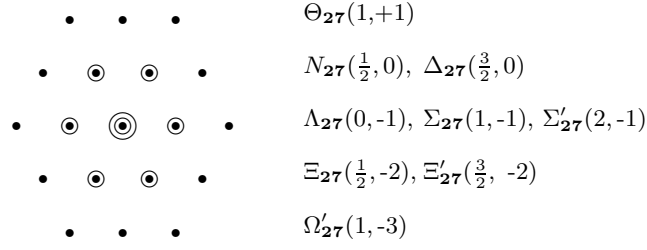
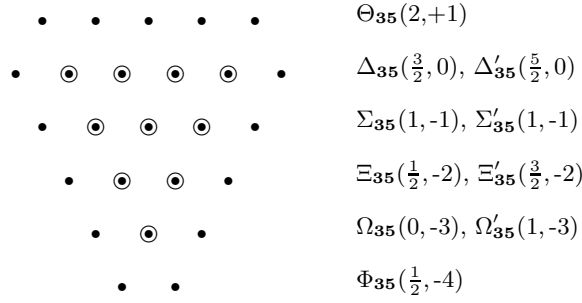


Figure 1: $SU(3)$ weight diagram of $\overline{10}$. Labels (I, S) to the right of a particle isomultiplet stand for isospin and strangeness.

Figure 2: $SU(3)$ weight diagram of $\mathbf{27}$.Figure 3: $SU(3)$ weight diagram of $\mathbf{35}$.APPENDIX B: DECAY AMPLITUDES OF $\bar{10}$ PENTAQUARK MULTIPLET

$\Theta_{\bar{10}}^+$	$\alpha_{\bar{10}}$	$\beta_{\bar{10}}$	$\gamma_{\bar{10}}$	$\delta_{\bar{10}}$	$\epsilon_{\bar{10}}$
PK^0	1	$-2\sqrt{3}$	0	0	0
NK^+	-1	$2\sqrt{3}$	0	0	0

$N_{\bar{10}}^+$	$\alpha_{\bar{10}}$	$\beta_{\bar{10}}$	$\gamma_{\bar{10}}$	$\delta_{\bar{10}}$	$\epsilon_{\bar{10}}$
$P\pi^0$	$-\sqrt{1/6}$	$\sqrt{1/2}$	0	$\sqrt{1/2}$	$-1/5\sqrt{6}$
$N\pi^+$	$-\sqrt{1/3}$	1	0	1	$-1/5\sqrt{3}$
$P\eta$	$\sqrt{1/2}$	$-\sqrt{3/2}$	$-\sqrt{2/3}$	$\sqrt{1/6}$	$-3/5\sqrt{2}$
ΛK^+	$-\sqrt{1/2}$	$\sqrt{3/2}$	$\sqrt{1/6}$	$-\sqrt{2/3}$	$-3/5\sqrt{2}$
$\Sigma^0 K^+$	$\sqrt{1/6}$	$-\sqrt{1/2}$	$\sqrt{1/2}$	0	$-1/5\sqrt{6}$
$\Sigma^+ K^0$	$\sqrt{1/3}$	-1	1	0	$-1/5\sqrt{3}$

$N_{\bar{10}}^0$	$\alpha_{\bar{10}}$	$\beta_{\bar{10}}$	$\gamma_{\bar{10}}$	$\delta_{\bar{10}}$	$\epsilon_{\bar{10}}$
$N\pi^0$	$-\sqrt{1/6}$	$\sqrt{1/2}$	0	$\sqrt{1/2}$	$-1/5\sqrt{6}$
$P\pi^-$	$\sqrt{1/3}$	-1	0	-1	$1/5\sqrt{3}$
$N\eta$	$-\sqrt{1/2}$	$\sqrt{3/2}$	$\sqrt{2/3}$	$-\sqrt{1/6}$	$3/5\sqrt{2}$
ΛK^0	$\sqrt{1/2}$	$-\sqrt{3/2}$	$-\sqrt{1/6}$	$\sqrt{2/3}$	$3/5\sqrt{2}$
$\Sigma^0 K^0$	$\sqrt{1/6}$	$-\sqrt{1/2}$	$\sqrt{1/2}$	0	$-1/5\sqrt{6}$
$\Sigma^- K^+$	$-\sqrt{1/3}$	1	-1	0	$1/5\sqrt{3}$

Σ_{10}^+	α_{10}	β_{10}	γ_{10}	δ_{10}	ϵ_{10}
$P\bar{K}^0$	$-\sqrt{1/3}$	0	1	0	$4/5\sqrt{3}$
$\Xi^0 K^+$	$\sqrt{1/3}$	0	0	1	$4/5\sqrt{3}$
$\Lambda\pi^+$	$-\sqrt{1/2}$	0	$\sqrt{1/6}$	$\sqrt{1/6}$	$-2\sqrt{2}/5$
$\Sigma^+\eta$	$\sqrt{1/2}$	0	$\sqrt{1/6}$	$\sqrt{1/6}$	$-2\sqrt{2}/5$
$\Sigma^0\pi^+$	$\sqrt{1/6}$	0	$\sqrt{1/2}$	$-\sqrt{1/2}$	0
$\Sigma^+\pi^0$	$-\sqrt{1/6}$	0	$-\sqrt{1/2}$	$\sqrt{1/2}$	0

Σ_{10}^0	α_{10}	β_{10}	γ_{10}	δ_{10}	ϵ_{10}
$\Lambda\pi^0$	$-\sqrt{1/2}$	0	$\sqrt{1/6}$	$\sqrt{1/6}$	$-2\sqrt{2}/5$
$\Sigma^0\eta$	$\sqrt{1/2}$	0	$\sqrt{1/6}$	$\sqrt{1/6}$	$-2\sqrt{2}/5$
$\Xi^0 K^0$	$-\sqrt{1/6}$	0	0	$-\sqrt{1/2}$	$-2\sqrt{2}/5\sqrt{3}$
$\Xi^- K^+$	$\sqrt{1/6}$	0	0	$\sqrt{1/2}$	$2\sqrt{2}/5\sqrt{3}$
$N\bar{K}^0$	$\sqrt{1/6}$	0	$-\sqrt{1/2}$	0	$-2\sqrt{2}/5\sqrt{3}$
PK^-	$-\sqrt{1/6}$	0	$\sqrt{1/2}$	0	$2\sqrt{2}/5\sqrt{3}$
$\Sigma^-\pi^+$	$-\sqrt{1/6}$	0	$-\sqrt{1/2}$	$\sqrt{1/2}$	0
$\Sigma^+\pi^-$	$\sqrt{1/6}$	0	$\sqrt{1/2}$	$-\sqrt{1/2}$	0

Σ_{10}^-	α_{10}	β_{10}	γ_{10}	δ_{10}	ϵ_{10}
NK^-	$\sqrt{1/3}$	0	-1	0	$-4/5\sqrt{3}$
$\Xi^- K^0$	$-\sqrt{1/3}$	0	0	-1	$-4/5\sqrt{3}$
$\Lambda\pi^-$	$\sqrt{1/2}$	0	$-\sqrt{1/6}$	$-\sqrt{1/6}$	$2\sqrt{2}/5$
$\Sigma^-\eta$	$-\sqrt{1/2}$	0	$-\sqrt{1/6}$	$-\sqrt{1/6}$	$2\sqrt{2}/5$
$\Sigma^0\pi^-$	$\sqrt{1/6}$	0	$\sqrt{1/2}$	$-\sqrt{1/2}$	0
$\Sigma^-\pi^0$	$-\sqrt{1/6}$	0	$-\sqrt{1/2}$	$\sqrt{1/2}$	0

Ξ_{10}^+	α_{10}	β_{10}	γ_{10}	δ_{10}	ϵ_{10}
$\Xi^0\pi^+$	1	$\sqrt{3}$	0	0	1
$\Sigma^+ K^0$	-1	$-\sqrt{3}$	0	0	1

Ξ_{10}^0	α_{10}	β_{10}	γ_{10}	δ_{10}	ϵ_{10}
$\Sigma^0 K^0$	$-\sqrt{2/3}$	$-\sqrt{2}$	0	0	$\sqrt{2/3}$
$\Sigma^+ K^-$	$-\sqrt{1/3}$	-1	0	0	$\sqrt{1/3}$
$\Xi^0\pi^0$	$\sqrt{2/3}$	$\sqrt{2}$	0	0	$\sqrt{2/3}$
$\Xi^-\pi^+$	$\sqrt{1/3}$	1	0	0	$\sqrt{1/3}$

Ξ_{10}^-	α_{10}	β_{10}	γ_{10}	δ_{10}	ϵ_{10}
$\Sigma^0 K^-$	$-\sqrt{2/3}$	$-\sqrt{2}$	0	0	$\sqrt{2/3}$
$\Sigma^- \bar{K}^0$	$\sqrt{1/3}$	1	0	0	$-\sqrt{1/3}$
$\Xi^0\pi^-$	$-\sqrt{1/3}$	-1	0	0	$-\sqrt{1/3}$
$\Xi^-\pi^0$	$\sqrt{2/3}$	$\sqrt{2}$	0	0	$\sqrt{2/3}$

Ξ_{10}^{--}	α_{10}	β_{10}	γ_{10}	δ_{10}	ϵ_{10}
$\Xi^-\pi^-$	-1	$-\sqrt{3}$	0	0	-1
$\Sigma^- K^-$	1	$\sqrt{3}$	0	0	-1

APPENDIX C: DECAY AMPLITUDES OF 27 PENTAQUARK MULTIPLET

Θ_{27}^{++}	α_{27}	β_{27}	γ_{27}	δ_{27}	ϵ_{27}	ζ_{27}	ψ_{27}
PK^+	$\sqrt{2}$	0	0	0	0	-8	4

Θ_{27}^+	α_{27}	β_{27}	γ_{27}	δ_{27}	ϵ_{27}	ζ_{27}	ψ_{27}
PK^0	1	0	0	0	0	$-4\sqrt{2}$	$2\sqrt{2}$
NK^+	1	0	0	0	0	$-4\sqrt{2}$	$2\sqrt{2}$

Θ_{27}^0	α_{27}	β_{27}	γ_{27}	δ_{27}	ϵ_{27}	ζ_{27}	ψ_{27}
NK^0	$\sqrt{2}$	0	0	0	0	-8	4

N_{27}^+	α_{27}	β_{27}	γ_{27}	δ_{27}	ϵ_{27}	ζ_{27}	ψ_{27}
$P\pi^0$	$\sqrt{1/30}$	0	$3/2$	0	$-\sqrt{3/5}$	$-14/5\sqrt{15}$	$1/5\sqrt{15}$
$N\pi^+$	$\sqrt{1/15}$	0	$3/\sqrt{2}$	0	$-\sqrt{6/5}$	$-14\sqrt{2}/5\sqrt{15}$	$\sqrt{2}/5\sqrt{15}$
$P\eta$	$3/\sqrt{10}$	$-\sqrt{3}$	$\sqrt{3}/2$	0	$3/\sqrt{5}$	$-42/5\sqrt{5}$	$3/5\sqrt{5}$
ΛK^+	$3/\sqrt{10}$	$\sqrt{3}/2$	$-\sqrt{3}$	0	$-3/\sqrt{5}$	$-42/5\sqrt{5}$	$3/5\sqrt{5}$
$\Sigma^0 K^+$	$\sqrt{1/30}$	$3/2$	0	0	$\sqrt{3/5}$	$-14/5\sqrt{15}$	$1/5\sqrt{15}$
$\Sigma^+ K^0$	$\sqrt{1/15}$	$3/\sqrt{2}$	0	0	$\sqrt{6/5}$	$-14\sqrt{2}/5\sqrt{15}$	$\sqrt{2}/5\sqrt{15}$

N_{27}^0	α_{27}	β_{27}	γ_{27}	δ_{27}	ϵ_{27}	ζ_{27}	ψ_{27}
$N\pi^0$	$-\sqrt{1/30}$	0	$-3/2$	0	$\sqrt{3/5}$	$14/5\sqrt{15}$	$-1/5\sqrt{15}$
$P\pi^-$	$\sqrt{1/15}$	0	$3/\sqrt{2}$	0	$-\sqrt{6/5}$	$-14\sqrt{2}/5\sqrt{15}$	$\sqrt{2}/5\sqrt{15}$
$N\eta$	$3/\sqrt{10}$	$-\sqrt{3}$	$\sqrt{3}/2$	0	$3/\sqrt{5}$	$-42/5\sqrt{5}$	$3/5\sqrt{5}$
ΛK^0	$3/\sqrt{10}$	$\sqrt{3}/2$	$-\sqrt{3}$	0	$-3/\sqrt{5}$	$-42/5\sqrt{5}$	$3/5\sqrt{5}$
$\Sigma^0 K^0$	$-\sqrt{1/30}$	$-3/2$	0	0	$-\sqrt{3/5}$	$14/5\sqrt{15}$	$-1/5\sqrt{15}$
$\Sigma^- K^+$	$\sqrt{1/15}$	$3/\sqrt{2}$	0	0	$\sqrt{6/5}$	$-14\sqrt{2}/5\sqrt{15}$	$\sqrt{2}/5\sqrt{15}$

Δ_{27}^{++}	α_{27}	β_{27}	γ_{27}	δ_{27}	ϵ_{27}	ζ_{27}	ψ_{27}
$P\pi^+$	1	0	0	$-3\sqrt{2}$	0	$-\sqrt{2}$	$2\sqrt{2}$
$\Sigma^+ K^+$	1	0	0	$3\sqrt{2}$	0	$-\sqrt{2}$	$2\sqrt{2}$

Δ_{27}^+	α_{27}	β_{27}	γ_{27}	δ_{27}	ϵ_{27}	ζ_{27}	ψ_{27}
$P\pi^0$	$\sqrt{2/3}$	0	0	$-2\sqrt{3}$	0	$-2/\sqrt{3}$	$4/\sqrt{3}$
$N\pi^+$	$-\sqrt{1/3}$	0	0	$\sqrt{6}$	0	$\sqrt{2/3}$	$-2\sqrt{2/3}$
$\Sigma^0 K^+$	$\sqrt{2/3}$	0	0	$2\sqrt{3}$	0	$-2/\sqrt{3}$	$4/\sqrt{3}$
$\Sigma^+ K^0$	$-\sqrt{1/3}$	0	0	$-\sqrt{6}$	0	$\sqrt{2/3}$	$-2\sqrt{2/3}$

Δ_{27}^0	α_{27}	β_{27}	γ_{27}	δ_{27}	ϵ_{27}	ζ_{27}	ψ_{27}
$P\pi^-$	$-\sqrt{1/3}$	0	0	$\sqrt{6}$	0	$\sqrt{2/3}$	$-2\sqrt{2/3}$
$N\pi^0$	$-\sqrt{2/3}$	0	0	$2\sqrt{3}$	0	$2/\sqrt{3}$	$-4/\sqrt{3}$
$\Sigma^0 K^0$	$-\sqrt{2/3}$	0	0	$-2\sqrt{3}$	0	$2/\sqrt{3}$	$-4/\sqrt{3}$
$\Sigma^- K^+$	$-\sqrt{1/3}$	0	0	$-\sqrt{6}$	0	$\sqrt{2/3}$	$-2\sqrt{2/3}$

Δ_{27}^-	α_{27}	β_{27}	γ_{27}	δ_{27}	ϵ_{27}	ζ_{27}	ψ_{27}
$N\pi^-$	1	0	0	$-3\sqrt{2}$	0	$-\sqrt{2}$	$2\sqrt{2}$
$\Sigma^- K^0$	1	0	0	$3\sqrt{2}$	0	$-\sqrt{2}$	$2\sqrt{2}$

$\Sigma_{27}^{'++}$	α_{27}	β_{27}	γ_{27}	δ_{27}	ϵ_{27}	ζ_{27}	ψ_{27}
$\Sigma^+ \pi^+$	$\sqrt{2}$	0	0	0	0	4	4

$\Sigma_{27}^{' +}$	α_{27}	β_{27}	γ_{27}	δ_{27}	ϵ_{27}	ζ_{27}	ψ_{27}
$\Sigma^0 \pi^+$	1	0	0	0	0	$2\sqrt{2}$	$2\sqrt{2}$
$\Sigma^+ \pi^0$	1	0	0	0	0	$2\sqrt{2}$	$2\sqrt{2}$

$\Sigma_{27}^{'0}$	α_{27}	β_{27}	γ_{27}	δ_{27}	ϵ_{27}	ζ_{27}	ψ_{27}
$\Sigma^0 \pi^0$	$2/\sqrt{3}$	0	0	0	0	$4\sqrt{2/3}$	$4\sqrt{2/3}$
$\Sigma^- \pi^+$	$-\sqrt{1/3}$	0	0	0	0	$-2\sqrt{2/3}$	$-2\sqrt{2/3}$
$\Sigma^+ \pi^-$	$-\sqrt{1/3}$	0	0	0	0	$-2\sqrt{2/3}$	$-2\sqrt{2/3}$

$\Sigma_{27}^{' -}$	α_{27}	β_{27}	γ_{27}	δ_{27}	ϵ_{27}	ζ_{27}	ψ_{27}
$\Sigma^0 \pi^-$	1	0	0	0	0	$2\sqrt{2}$	$2\sqrt{2}$
$\Sigma^- \pi^0$	1	0	0	0	0	$2\sqrt{2}$	$2\sqrt{2}$

$\Sigma_{27}^{' --}$	α_{27}	β_{27}	γ_{27}	δ_{27}	ϵ_{27}	ζ_{27}	ψ_{27}
$\Sigma^- \pi^-$	$\sqrt{2}$	0	0	0	0	4	4

Σ_{27}^+	α_{27}	β_{27}	γ_{27}	δ_{27}	ϵ_{27}	ζ_{27}	ψ_{27}
PK^0	$-\sqrt{2/5}$	$\sqrt{3}$	0	$4/\sqrt{5}$	$-4/\sqrt{5}$	$4/5\sqrt{5}$	$4/5\sqrt{5}$
$\Xi^0 K^+$	$-\sqrt{2/5}$	0	$\sqrt{3}$	$-4/\sqrt{5}$	$4/\sqrt{5}$	$4/5\sqrt{5}$	$4/5\sqrt{5}$
$\Lambda\pi^+$	$\sqrt{3/5}$	$\sqrt{1/2}$	$\sqrt{1/2}$	$-2\sqrt{6/5}$	$-2\sqrt{6/5}$	$-2\sqrt{6/5}\sqrt{5}$	$-2\sqrt{6/5}\sqrt{5}$
$\Sigma^+\eta$	$\sqrt{3/5}$	$\sqrt{1/2}$	$\sqrt{1/2}$	$2\sqrt{6/5}$	$2\sqrt{6/5}$	$-2\sqrt{6/5}\sqrt{5}$	$-2\sqrt{6/5}\sqrt{5}$
$\Sigma^0\pi^+$	0	$\sqrt{3/2}$	$-\sqrt{3/2}$	$-2\sqrt{2/5}$	$2\sqrt{2/5}$	0	0
$\Sigma^+\pi^0$	0	$-\sqrt{3/2}$	$\sqrt{3/2}$	$2\sqrt{2/5}$	$-2\sqrt{2/5}$	0	0

Σ_{27}^0	α_{27}	β_{27}	γ_{27}	δ_{27}	ϵ_{27}	ζ_{27}	ψ_{27}
$\Lambda\pi^0$	$-\sqrt{3/5}$	$-\sqrt{1/2}$	$-\sqrt{1/2}$	$2\sqrt{6/5}$	$2\sqrt{6/5}$	$2\sqrt{6/5}\sqrt{5}$	$2\sqrt{6/5}\sqrt{5}$
$\Sigma^0\eta$	$-\sqrt{3/5}$	$-\sqrt{1/2}$	$-\sqrt{1/2}$	$-2\sqrt{6/5}$	$-2\sqrt{6/5}$	$2\sqrt{6/5}\sqrt{5}$	$2\sqrt{6/5}\sqrt{5}$
$N\bar{K}^0$	$-\sqrt{1/5}$	$\sqrt{3/2}$	0	$2\sqrt{2/5}$	$-2\sqrt{2/5}$	$2\sqrt{2/5}\sqrt{5}$	$2\sqrt{2/5}\sqrt{5}$
PK^-	$\sqrt{1/5}$	$-\sqrt{3/2}$	0	$-2\sqrt{2/5}$	$2\sqrt{2/5}$	$-2\sqrt{2/5}\sqrt{5}$	$-2\sqrt{2/5}\sqrt{5}$
$\Xi^- K^+$	$\sqrt{1/5}$	0	$-\sqrt{3/2}$	$2\sqrt{2/5}$	$-2\sqrt{2/5}$	$-2\sqrt{2/5}\sqrt{5}$	$-2\sqrt{2/5}\sqrt{5}$
$\Xi^0 K^0$	$-\sqrt{1/5}$	0	$\sqrt{3/2}$	$-2\sqrt{2/5}$	$2\sqrt{2/5}$	$2\sqrt{2/5}\sqrt{5}$	$2\sqrt{2/5}\sqrt{5}$
$\Sigma^-\pi^+$	0	$\sqrt{3/2}$	$-\sqrt{3/2}$	$-2\sqrt{2/5}$	$2\sqrt{2/5}$	0	0
$\Sigma^+\pi^-$	0	$-\sqrt{3/2}$	$\sqrt{3/2}$	$2\sqrt{2/5}$	$-2\sqrt{2/5}$	0	0

Σ_{27}^-	α_{27}	β_{27}	γ_{27}	δ_{27}	ϵ_{27}	ζ_{27}	ψ_{27}
NK^-	$-\sqrt{2/5}$	$\sqrt{3}$	0	$4/\sqrt{5}$	$-4/\sqrt{5}$	$4/5\sqrt{5}$	$4/5\sqrt{5}$
$\Xi^- K^0$	$-\sqrt{2/5}$	0	$\sqrt{3}$	$-4/\sqrt{5}$	$4/\sqrt{5}$	$4/5\sqrt{5}$	$4/5\sqrt{5}$
$\Lambda\pi^-$	$\sqrt{3/5}$	$\sqrt{1/2}$	$\sqrt{1/2}$	$-2\sqrt{6/5}$	$-2\sqrt{6/5}$	$-2\sqrt{6/5}\sqrt{5}$	$-2\sqrt{6/5}\sqrt{5}$
$\Sigma^-\eta$	$\sqrt{3/5}$	$\sqrt{1/2}$	$\sqrt{1/2}$	$2\sqrt{6/5}$	$2\sqrt{6/5}$	$-2\sqrt{6/5}\sqrt{5}$	$-2\sqrt{6/5}\sqrt{5}$
$\Sigma^0\pi^-$	0	$-\sqrt{3/2}$	$\sqrt{3/2}$	$2\sqrt{2/5}$	$-2\sqrt{2/5}$	0	0
$\Sigma^-\pi^0$	0	$\sqrt{3/2}$	$-\sqrt{3/2}$	$-2\sqrt{2/5}$	$2\sqrt{2/5}$	0	0

Λ_{27}	α_{27}	β_{27}	γ_{27}	δ_{27}	ϵ_{27}	ζ_{27}	ψ_{27}
$N\bar{K}^0$	$\sqrt{3/2}\sqrt{5}$	$-3/2\sqrt{2}$	$3/\sqrt{2}$	0	0	$-4\sqrt{6/5}\sqrt{5}$	$-4\sqrt{6/5}\sqrt{5}$
PK^-	$\sqrt{3/2}\sqrt{5}$	$-3/2\sqrt{2}$	$3/\sqrt{2}$	0	0	$-4\sqrt{6/5}\sqrt{5}$	$-4\sqrt{6/5}\sqrt{5}$
$\Lambda\eta$	$-3\sqrt{3/2}\sqrt{5}$	$3/2\sqrt{2}$	$3/2\sqrt{2}$	0	0	$12\sqrt{6/5}\sqrt{5}$	$12\sqrt{6/5}\sqrt{5}$
$\Sigma^-\pi^+$	$-1/2\sqrt{15}$	$-3/2\sqrt{2}$	$-3/2\sqrt{2}$	0	0	$4\sqrt{2/5}\sqrt{15}$	$4\sqrt{2/5}\sqrt{15}$
$\Sigma^0\pi^0$	$-1/2\sqrt{15}$	$-3/2\sqrt{2}$	$-3/2\sqrt{2}$	0	0	$4\sqrt{2/5}\sqrt{15}$	$4\sqrt{2/5}\sqrt{15}$
$\Sigma^+\pi^-$	$-1/2\sqrt{15}$	$-3/2\sqrt{2}$	$-3/2\sqrt{2}$	0	0	$4\sqrt{2/5}\sqrt{15}$	$4\sqrt{2/5}\sqrt{15}$
$\Xi^- K^+$	$\sqrt{3/2}\sqrt{5}$	$3/\sqrt{2}$	$-3/2\sqrt{2}$	0	0	$-4\sqrt{6/5}\sqrt{5}$	$-4\sqrt{6/5}\sqrt{5}$
$\Xi^0 K^0$	$\sqrt{3/2}\sqrt{5}$	$3/\sqrt{2}$	$-3/2\sqrt{2}$	0	0	$-4\sqrt{6/5}\sqrt{5}$	$-4\sqrt{6/5}\sqrt{5}$

Ξ_{27}^+	α_{27}	β_{27}	γ_{27}	δ_{27}	ϵ_{27}	ζ_{27}	ψ_{27}
$\Xi^0\pi^+$	1	0	0	0	$-3\sqrt{2}$	$2\sqrt{2}$	$-\sqrt{2}$
$\Sigma^+\bar{K}^0$	1	0	0	0	$3\sqrt{2}$	$2\sqrt{2}$	$-\sqrt{2}$

Ξ_{27}^0	α_{27}	β_{27}	γ_{27}	δ_{27}	ϵ_{27}	ζ_{27}	ψ_{27}
$\Sigma^+ K^-$	$\sqrt{1/3}$	0	0	0	$\sqrt{6}$	$2\sqrt{2/3}$	$-\sqrt{2/3}$
$\Sigma^0\bar{K}^0$	$\sqrt{2/3}$	0	0	0	$2\sqrt{3}$	$4/\sqrt{3}$	$-2/\sqrt{3}$
$\Xi^0\pi^0$	$\sqrt{2/3}$	0	0	0	$-2\sqrt{3}$	$4/\sqrt{3}$	$-2/\sqrt{3}$
$\Xi^-\pi^+$	$\sqrt{1/3}$	0	0	0	$-\sqrt{6}$	$2\sqrt{2/3}$	$-\sqrt{2/3}$

$\Xi_{27}^{'-}$	α_{27}	β_{27}	γ_{27}	δ_{27}	ϵ_{27}	ζ_{27}	ψ_{27}
$\Sigma^-\bar{K}^0$	$-\sqrt{1/3}$	0	0	0	$-\sqrt{6}$	$-2\sqrt{2/3}$	$\sqrt{2/3}$
$\Sigma^0 K^-$	$\sqrt{2/3}$	0	0	0	$2\sqrt{3}$	$4/\sqrt{3}$	$-2/\sqrt{3}$
$\Xi^-\pi^0$	$\sqrt{2/3}$	0	0	0	$-2\sqrt{3}$	$4/\sqrt{3}$	$-2/\sqrt{3}$
$\Xi^0\pi^-$	$-\sqrt{1/3}$	0	0	0	$\sqrt{6}$	$-2\sqrt{2/3}$	$\sqrt{2/3}$

$\Xi_{27}^{'--}$	α_{27}	β_{27}	γ_{27}	δ_{27}	ϵ_{27}	ζ_{27}	ψ_{27}
$\Xi^-\pi^-$	1	0	0	0	$-3\sqrt{2}$	$2\sqrt{2}$	$-\sqrt{2}$
$\Sigma^- K^-$	1	0	0	0	$3\sqrt{2}$	$2\sqrt{2}$	$-\sqrt{2}$

Ξ_{27}^0	α_{27}	β_{27}	γ_{27}	δ_{27}	ϵ_{27}	ζ_{27}	ψ_{27}
$\Xi^0 \pi^0$	$-\sqrt{1/30}$	$-3/2$	0	$\sqrt{3/5}$	0	$-1/5\sqrt{15}$	$14/5\sqrt{15}$
$\Xi^- \pi^+$	$\sqrt{1/15}$	$3/\sqrt{2}$	0	$-\sqrt{6/5}$	0	$\sqrt{2/5}\sqrt{15}$	$-14\sqrt{2/5}\sqrt{15}$
$\Xi^0 \eta$	$3/\sqrt{10}$	$\sqrt{3}/2$	$-\sqrt{3}$	$3/\sqrt{5}$	0	$3/5\sqrt{5}$	$-42/5\sqrt{5}$
$\Lambda \bar{K}^0$	$3/\sqrt{10}$	$-\sqrt{3}$	$\sqrt{3}/2$	$-3/\sqrt{5}$	0	$3/5\sqrt{5}$	$-42/5\sqrt{5}$
$\Sigma^0 \bar{K}^0$	$-\sqrt{1/30}$	0	$-3/2$	$-\sqrt{3/5}$	0	$-1/5\sqrt{15}$	$14/5\sqrt{15}$
$\Sigma^+ K^-$	$\sqrt{1/15}$	0	$3/\sqrt{2}$	$\sqrt{6/5}$	0	$\sqrt{2/5}\sqrt{15}$	$-14\sqrt{2/5}\sqrt{15}$

Ξ_{27}^-	α_{27}	β_{27}	γ_{27}	δ_{27}	ϵ_{27}	ζ_{27}	ψ_{27}
$\Xi^- \pi^0$	$\sqrt{1/30}$	$3/2$	0	$-\sqrt{3/5}$	0	$1/5\sqrt{15}$	$-14/5\sqrt{15}$
$\Xi^0 \pi^-$	$\sqrt{1/15}$	$3/\sqrt{2}$	0	$-\sqrt{6/5}$	0	$\sqrt{2/5}\sqrt{15}$	$-14\sqrt{2/5}\sqrt{15}$
$\Xi^- \eta$	$3/\sqrt{10}$	$\sqrt{3}/2$	$-\sqrt{3}$	$3/\sqrt{5}$	0	$3/5\sqrt{5}$	$-42/5\sqrt{5}$
ΛK^-	$3/\sqrt{10}$	$-\sqrt{3}$	$\sqrt{3}/2$	$-3/\sqrt{5}$	0	$3/5\sqrt{5}$	$-42/5\sqrt{5}$
$\Sigma^0 K^-$	$\sqrt{1/30}$	0	$3/2$	$\sqrt{3/5}$	0	$1/5\sqrt{15}$	$-14/5\sqrt{15}$
$\Sigma^- \bar{K}^0$	$\sqrt{1/15}$	0	$3/\sqrt{2}$	$\sqrt{6/5}$	0	$\sqrt{2/5}\sqrt{15}$	$-14\sqrt{2/5}\sqrt{15}$

Ω_{27}^0	α_{27}	β_{27}	γ_{27}	δ_{27}	ϵ_{27}	ζ_{27}	ψ_{27}
$\Xi^0 \bar{K}^0$	$\sqrt{2}$	0	0	0	0	4	-8

Ω_{27}^-	α_{27}	β_{27}	γ_{27}	δ_{27}	ϵ_{27}	ζ_{27}	ψ_{27}
$\Xi^0 K^-$	1	0	0	0	0	$2\sqrt{2}$	$-4\sqrt{2}$
$\Xi^- \bar{K}^0$	1	0	0	0	0	$2\sqrt{2}$	$-4\sqrt{2}$

Ω_{27}^{--}	α_{27}	β_{27}	γ_{27}	δ_{27}	ϵ_{27}	ζ_{27}	ψ_{27}
$\Xi^- K^-$	$\sqrt{2}$	0	0	0	0	4	-8

APPENDIX D: DECAY AMPLITUDES OF 35 PENTAQUARK MULTIPLET

$\Theta_{35}^-, \Theta_{35}^0, \Theta_{35}^+, \Theta_{35}^{++}, \Theta_{35}^{+++}$	β_{35}	γ_{35}
Stable	0	0

$\Delta_{35}'^{+++}, \Delta_{35}'^{++}, \Delta_{35}'^+, \Delta_{35}'^0, \Delta_{35}'^-, \Delta_{35}'^{--}$	β_{35}	γ_{35}
Stable	0	0

Δ_{35}^{++}	β_{35}	γ_{35}
$P\pi^+$	-1	1
$\Sigma^+ K^+$	1	1

Δ_{35}^+	β_{35}	γ_{35}
$P\pi^0$	$\sqrt{2/3}$	$-\sqrt{2/3}$
$N\pi^+$	$-\sqrt{1/3}$	$\sqrt{1/3}$
$\Sigma^0 K^+$	$-\sqrt{2/3}$	$-\sqrt{2/3}$
$\Sigma^+ K^0$	$\sqrt{1/3}$	$\sqrt{1/3}$

Δ_{35}^0	β_{35}	γ_{35}
$P\pi^-$	$\sqrt{1/3}$	$-\sqrt{1/3}$
$N\pi^0$	$\sqrt{2/3}$	$-\sqrt{2/3}$
$\Sigma^- K^+$	$-\sqrt{1/3}$	$-\sqrt{1/3}$
$\Sigma^0 K^0$	$-\sqrt{2/3}$	$-\sqrt{2/3}$

Δ_{35}^-	β_{35}	γ_{35}
$N\pi^-$	1	-1
$\Sigma^- K^0$	-1	-1

$\Sigma_{35}'^{++}$	β_{35}	γ_{35}
$\Sigma^+ \pi^+$	0	$-2\sqrt{6/5}$

Σ'_{35}^+	β_{35}	γ_{35}
$\Sigma^0 \pi^+$	0	$-2\sqrt{3/5}$
$\Sigma^+ \pi^0$	0	$-2\sqrt{3/5}$

$\Sigma'_{35}{}^0$	β_{35}	γ_{35}
$\Sigma^0 \pi^0$	0	$4/\sqrt{5}$
$\Sigma^- \pi^+$	0	$-2/\sqrt{5}$
$\Sigma^+ \pi^-$	0	$-2/\sqrt{5}$

$\Sigma'_{35}{}^-$	β_{35}	γ_{35}
$\Sigma^0 \pi^-$	0	$2\sqrt{3/5}$
$\Sigma^- \pi^0$	0	$2\sqrt{3/5}$

$\Sigma'_{35}{}^{--}$	β_{35}	γ_{35}
$\Sigma^- \pi^-$	0	$2\sqrt{6/5}$

Σ_{35}^+	β_{35}	γ_{35}
$P\bar{K}^0$	$-2\sqrt{2/15}$	$2\sqrt{2/15}$
$\Xi^0 K^+$	$2\sqrt{2/15}$	$2\sqrt{2/15}$
$\Lambda \pi^+$	$2/\sqrt{5}$	$-2/\sqrt{5}$
$\Sigma^0 \pi^+$	$2/\sqrt{15}$	0
$\Sigma^+ \pi^0$	$-2/\sqrt{15}$	0
$\Sigma^+ \eta$	$-2/\sqrt{5}$	$-2/\sqrt{5}$

Σ_{35}^0	β_{35}	γ_{35}
$N\bar{K}^0$	$-2/\sqrt{15}$	$2/\sqrt{15}$
$P\bar{K}^-$	$2/\sqrt{15}$	$-2/\sqrt{15}$
$\Lambda \pi^0$	$-2/\sqrt{5}$	$2/\sqrt{5}$
$\Sigma^- \pi^+$	$2/\sqrt{15}$	0
$\Sigma^+ \pi^-$	$-2/\sqrt{15}$	0
$\Sigma^0 \eta$	$2/\sqrt{5}$	$2/\sqrt{5}$
$\Xi^0 K^0$	$2/\sqrt{15}$	$2/\sqrt{15}$
$\Xi^- K^+$	$-2/\sqrt{15}$	$-2/\sqrt{15}$

Σ_{35}^-	β_{35}	γ_{35}
NK^-	$2\sqrt{2/15}$	$-2\sqrt{2/15}$
$\Xi^- K^0$	$-2\sqrt{2/15}$	$-2/\sqrt{2/15}$
$\Lambda \pi^-$	$-2/\sqrt{5}$	$2/\sqrt{5}$
$\Sigma^- \pi^0$	$-2/\sqrt{15}$	0
$\Sigma^0 \pi^-$	$2/\sqrt{15}$	0
$\Sigma^- \eta$	$2/\sqrt{5}$	$2/\sqrt{5}$

Ξ'_{35}^+	β_{35}	γ_{35}
$\Xi^0 \pi^+$	0	$-4/\sqrt{5}$
$\Sigma^+ \bar{K}^0$	0	$-4/\sqrt{5}$

$\Xi'_{35}{}^0$	β_{35}	γ_{35}
$\Xi^- \pi^+$	0	$-4/\sqrt{15}$
$\Xi^0 \pi^0$	0	$-4\sqrt{2/15}$
$\Sigma^+ K^-$	0	$-4/\sqrt{15}$
$\Sigma^0 \bar{K}^0$	0	$-4\sqrt{2/15}$

$\Xi'_{35}{}^-$	β_{35}	γ_{35}
$\Xi^0 \pi^-$	0	$-4/\sqrt{15}$
$\Xi^- \pi^0$	0	$4\sqrt{2/15}$
$\Sigma^- \bar{K}^0$	0	$-4/\sqrt{15}$
$\Sigma^0 K^-$	0	$4\sqrt{2/15}$

$\Xi_{35}'^{--}$	β_{35}	γ_{35}
$\Xi^- \pi^-$	0	$-4/\sqrt{5}$
$\Sigma^- K^-$	0	$-4/\sqrt{5}$

Ξ_{35}^0	β_{35}	γ_{35}
$\Xi^0 \pi^0$	$-\sqrt{3/10}$	$\sqrt{1/30}$
$\Xi^- \pi^+$	$\sqrt{3/5}$	$-\sqrt{1/15}$
$\Xi^0 \eta$	$-3/\sqrt{10}$	$-3/\sqrt{10}$
ΛK^0	$3/\sqrt{10}$	$-3/\sqrt{10}$
$\Sigma^0 \bar{K}^0$	$\sqrt{3/10}$	$\sqrt{1/30}$
$\Sigma^+ K^-$	$-\sqrt{3/5}$	$-\sqrt{1/15}$

Ξ_{35}^-	β_{35}	γ_{35}
$\Xi^- \pi^0$	$-\sqrt{3/10}$	$\sqrt{1/30}$
$\Xi^0 \pi^-$	$-\sqrt{3/5}$	$\sqrt{1/15}$
$\Xi^- \eta$	$3/\sqrt{10}$	$3/\sqrt{10}$
ΛK^-	$-3/\sqrt{10}$	$3/\sqrt{10}$
$\Sigma^0 K^-$	$\sqrt{3/10}$	$\sqrt{1/30}$
$\Sigma^- \bar{K}^0$	$\sqrt{3/5}$	$\sqrt{1/15}$

$\Omega_{35}^{'0}$	β_{35}	γ_{35}
$\Xi^0 \bar{K}^0$	0	$-2/\sqrt{6/5}$

$\Omega_{35}^{'-}$	β_{35}	γ_{35}
$\Xi^0 K^-$	0	$-2\sqrt{3/5}$
$\Xi^- \bar{K}^0$	0	$-2\sqrt{3/5}$

$\Omega_{35}^{'--}$	β_{35}	γ_{35}
$\Xi^- K^-$	0	$2/\sqrt{6/5}$

Ω_{35}^-	β_{35}	γ_{35}
$\Xi^0 K^-$	$-2\sqrt{2/5}$	0
$\Xi^- \bar{K}^0$	$2\sqrt{2/5}$	0

Φ_{35}^-, Φ_{35}^0	β_{35}	γ_{35}
Stable	0	0